

# Optimal scheduling of manufacturing processes across multiple production lines by polynomial regression and bagged bounded binary knapsack

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**Abstract** The extensive sensorisation of manufacturing operations in ‘Industry 4.0’ implies the possibility of pursuing aggressive scheduling optimisations across multiple lines. The present contribution introduces an attempt toward this objective that focuses methodologically on polynomial regression and bagged bounded binary knapsack. Results are evaluated on real data from a sensorised metal injection molding plant.

**Keywords** optimization, scheduling

## 1 Introduction

The understanding of industrial production processes and their optimization represent valuable objectives for the effective management of manufacturing plants. Real time collection of sensor data represents a valuable instrument to this end. On-line analytical procedures can be set up to summarize the state of the production plant and to identify and suggest possibilities for improvement in the overall efficiency. The global optimization of the scheduling of production across multiple manufacturing lines is an area showing significant promise with this respect. The present contribution introduces the results of a specific study carried out on real data resulting in the definition, implementation and validation of procedures for the optimal scheduling of production across a manufacturing plant operating by metal injection molding. Furthermore, the on-line capabilities of

the proposed scheduling system imply that unforeseen contingency situations related e.g. to the interruption of manufacturing processes in one or more lines can be accounted for by rapidly rescheduling the production of the required output on available facilities. The present work builds on contributions put forward in the state of the art by a number of authors proposing related methodologies.

## 2 Energy Optimization

For  $M$  given machines, an energy-optimal production task schedule can be defined based on linear programming methods based on operations research. An objective function  $O_i(\cdot)$  specific to the production of piece  $i$  can be defined based on the total number of pieces produced. If  $n_m$  is the number of pieces per hour that a specific machine  $m$  is capable of producing,  $t_m$  is the time that the machine needs to produce each piece and  $e_m$  is power consumption in  $W$  for the given injector per hour, then obviously:

$$O_i = \sum_{m=0}^M \sum_{i=0}^I (n_i^m \cdot t_i^m) \cdot e_i^m \quad (1)$$

The objective function can easily be numerically optimised by linear programming methods based on polynomial regression. An example optimized schedule is represented in Figure 1.

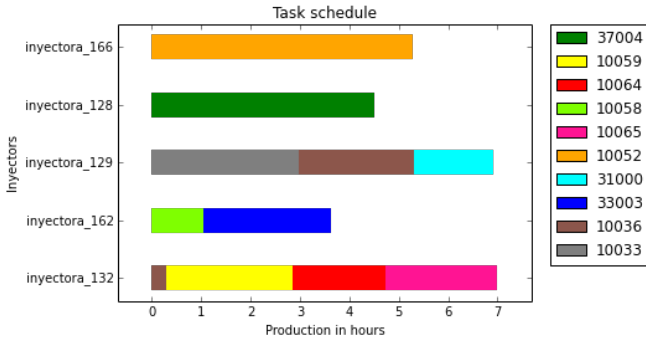
## 3 Time optimization

While energy optimization can easily be carried out by linear methods, a further objective function describing

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**Fig. 1** Cumulative time in hours for the production of a specific piece in different injectors - Energy optimal scheduling.

the time required for overall production can only be described by a non-linear expression of the kind of

$$T = \max \sum_{i=0}^I t_i^m \quad (2)$$

whose optimization can be pursued by dynamic programming methods. Bagged binary knapsack can be exploited to this end, mapping the local processing times in the optimal scheduling problem to the item weights in the standard formulation of the problem. An approximation based on the Dantzig method can be employed to greedily solve the knapsack problem, sorting the items in decreasing order of value per unit of weight and inserting them in order in the current proposed solution set. The obtained methodology represents a heuristic converging to a local minimum. If  $T^m$  represents the slack for given machine  $m$  then  $(\sum_m v_p^m \cdot T^m)_{p \in P}$  is the maximum number of produceable pieces considering all machines concurrently. A valid scheduling solution must verify the condition that the scheduling order it represents does not exceed production capacity:

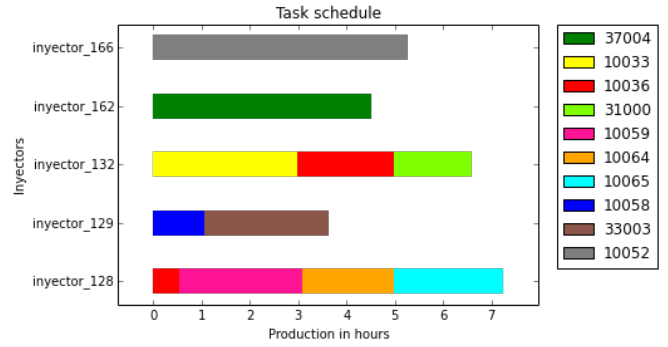
$$O_p \leq \left( \sum_m v_p^m \cdot T^m \right)_{p \in P} \quad \forall p \in P.$$

If we consider the symbols

- available machine time  $T^m$  corresponding to the slack for  $m$  as the knapsack to be filled
- urgency  $O_p / \sum_m v_p^m t^m$  as the value of the item to be put in the knapsack
- max produceable pieces  $v_p^m t^m$  as number of available items
- weight as the number of minutes to produce the product  $1/v_p^m$

then the most urgent product  $\hat{p}$  can be identified by

$$\hat{p} = \arg \max_p \left( \frac{O_p}{\sum_m v_p^m \cdot T^m} \right)_p.$$



**Fig. 2** Cumulative time in hours for the production of a specific piece in different injectors - Time optimal scheduling.

The machine  $\hat{m}_{\hat{p}}$  that is most available for it

$$\hat{m}_{\hat{p}} = \arg \max_m (v_{p=\hat{p}}^m \cdot T^m)_m$$

can be assigned to it to progressively allocate the available time to pieces to be produced.

Equivalent rescaled and quantized weights for the machine that is fastest at the most urgent product can be described as

$$(w_p^{m=\hat{m}_{\hat{p}}})_{p \in P} = (1/v_p^{m=\hat{m}_{\hat{p}}})_{p \in P}.$$

Iteratively updating the order matrix

$$\sum_p \hat{T}_p^{m=\hat{m}} = \sum_p v_p^{\hat{m}} / \hat{O}_p^{\hat{m}}$$

and the computed slack by  $T^m - \sum_p \hat{T}_p^m$  does result in an effective optimization of the production schedule across production lines that is attainable in a limited number of iterations and is therefore usable for on-line scheduling of manufacturing orders (figure 2).

## 4 Conclusions

The sensorisation of production lines permits production scheduling optimisation across multiple production lines. We present an attempt toward this objective that focuses methodologically on polynomial regression and bagged binary knapsack. The methodology is demonstrated and evaluated on real data from a sensorized manufacturing plant operating by injection-based procedures.